

g the last several years, many scientists have turned the world around us to unravel the complex web of interconnections that characterize seemingly all social [1], biological [2, 3] and technological systems [5]. These systems have been shown to exhibit features that can be captured using the tools of network theory or in more recent terms, network modeling. At the same time, network models of diverse kinds have been proposed with the aim of describing and explaining the properties of real webs [6, 7]. It turns out that most networks are better described by growing models in which the number of nodes (or elements) forming the network increases with time and that the probability that a given node has k connections to other nodes follows a power-law distribution, $\sim k^{-\gamma}$, with $\gamma \leq 3$. Additionally, the study of processes taking place on top of these networks has led to consider classical results obtained for regular lattices and random graphs due to the radical changes of the dynamical properties when the heterogeneity of complex networks can not be neglected [8, 9, 10, 11].

The first scale-free network model, introduced by Barabási and Albert (BA), postulated that there are two fundamental ingredients of many real networks [12, 13]: the growing character and the preferential attachment rule. The main feature of the BA model is that it is

arbitrary γ -exponents, and non-random correlations can be found nowadays in the scientific literature. On the other hand, there are some models in which the PA rule is limited to a neighborhood due to geographic constraints [14], or where its linear character is investigated [15]. Recently, Caldarelli *et al.* [16] have shown that some models produce SF networks without assuming preferential attachment at all. As a byproduct, other properties of the network fit well with those of real-world graphs. They introduced an intrinsic fitness model in which the nodes are connected with a probability that depends on their fitness. Note, additionally, that the way in which the fitness parameter was introduced is different from that in the model in [17].

In this paper, we adopt a different perspective. The main aim is to test to what extend the global character of the PA rule in the original BA model is important. We propose to introduce a model in which the PA is applied only to the neighborhood of the newly added node depending on the value of a variable which measures the affinity between the two adjacent nodes. By going down from the BA limit of the model to the limit where all nodes are distinct, we can test to what extend the global knowledge of each node's activity is fundamental to get a scale-free graph. The main conclusion is that the introduction of a

$$\Pi(k_i) = \frac{k_i}{\sum_{s \in A} k_s} \quad (1)$$

ally v) Repeat steps *ii-iv* such that the final size network is $N = m_o + t$.

after t time steps a network made up of N nodes. It is worth mentioning that the inclusion of the parameter a is not a mere artifact. Indeed, most systems are formed by non-identical elements and it is natural to assume that although a given node have a large connectivity a newly created element link to that node because they have very little in common. This feature is clearly manifested in social systems like the WWW –where individuals bookmark

web pages accordingly to their “affinity” – or the scientist citation network [19]. In this way, it is very easy to find a citation in a condensed matter paper belonging to a paper wrote by a psychologist. Additionally, the same argument can be translated to biological systems such as predator-prey webs or protein-protein interaction networks.

Finally, when μ is large enough as to dilute the first connections of the model, we recover the BA model. The problem consists of determining to what extent the local preferential attachment will give the same results, or in other words, does the knowledge of the entire network potentially contribute to the properties observed in the

FIG. 1: Number of nodes with connectivity k for different values of μ . The size of the network is $N = 10^4$ nodes, $m_o = m = 3$. The power-law distribution has an exponent equal to 3. Note that the BA limit corresponds to

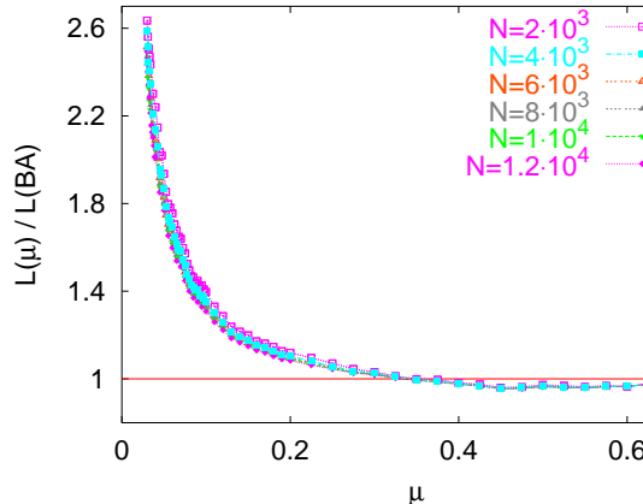


FIG. 2: Ratio between the average shortest path length for different μ values, $L(\mu)$, and that of the BA network for several system sizes. The horizontal line marks the BA limit. A transition from graphs fulfilling the small-world properties to a regime in which networks break down in many small clusters is observed as the value of $L(\mu)$ is rising. See the text for more details.

work is made up of $N = 500$ nodes.

FIG. 4: Average clustering coefficient c_k of nodes with k for five different values of the parameter μ . Note that as μ decreases, the clustering coefficient departs from the limit ($\mu = 1$). The parameters used for the generated networks are as of fig. 1.

es, but there is nothing that guarantees *a priori* that components of the network will link together in a way that other properties will not be affected. In the case of the average shortest path length L , the average shortest path length of a graph is defined as the minimum number of nodes one has to pass by to go from one node of the network to another randomly chosen node, averaged over all possible pairs of nodes. Complex networks show the noticeable property, known as small-world property, that the average path length increases with the logarithm of its size. We expect that for values of μ the network is composed by a unique component and no fragmentation arises. When the degree to which the affinity criterion is applied decreases, the network will gradually loose its compactness and stretch approaching a one-dimensional structure with one small components. Further reduction of μ produces a break down of the network in many isolated

efficient c_i . The clustering coefficient of a node is defined as the ratio between the number of edges between the k_i neighbors of i and its maximum possible value $k_i(k_i - 1)/2$, i.e., $c_i = \frac{2e_i}{k_i(k_i - 1)}$. In this way, the clustering coefficient, c is given by the average of all nodes of the network. The clustering coefficient has a local character as it gives the probability that two nodes with a common neighbor are also linked together. It is expected that this magnitude, in our model, depends on the affinity of each node and the range of probability of attachment given by μ . Figure 4 shows the average clustering coefficient of nodes with a given connectivity for different values of the parameter μ . The figure exhibits almost no correlations with the degree of vertices and the smallest value for the clustering coefficient. As μ is reduced, the first selection of nodes with high affinity values plays a more dominant role compared to the rising of c_i for small and large connectivity. At the transition, $\mu \sim 0.04$, the average coefficient is one order of magnitude greater than that of the

work

Figure 2 and 3 substantiate this picture. Figure 2 presents the ratio between the average path length obtained for different values of μ and that of the BA network for several system sizes. As μ restricts the PA network undergoes a transition characterized by a

Average nearest neighbor connectivity k_{nn} against k values of μ . Results are averaged over 100 network runs for each μ value. Other parameters are as of fig.

the tendency that networks generated with small μ display disassortative mixing at both ends of connectivity range.

In this paper, we have studied a version of the Barabási-Albert scale-free model that allows to tune the range in which the preferential attachment is applied. The model considers that all nodes are different such that they are in principle unable to link to very distinct nodes. By applying an affinity selection before applying the preferential attachment rule, we tested whether or not the range of the entire network is an essential requisite for scale-free networks. Our results seem to support the idea that having at least some degree of preferential

connectivity is important for the emergence of scale-free properties. In this sense, it would be interesting to perform the same analysis in more realistic growing network models looking for more similarities with real-world networks. For example, the exponent of the connectivity distribution can be tuned to smaller values by incorporating the first level of selection of the model in the generalized BA model [6], which allows to give arbitrary γ values in the interval $(2, 3)$.

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